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DISCUSSION.

PROFESSOR FULLERTON'S DOCTRINE OF SPACE.

THE recent articles in the PHILOSOPHICAL REVIEW (Mch.–Nov., 1901) by Professor Fullerton on the time and space problem are characterized by a point of view that demands further examination. They present an elaborate demonstration of the absurdities of the Kantian doctrine, and offer in its stead a theory which Professor Fullerton calls the Berkeleian. Aside from some minor contentions and quibbles, the gist of the difficulty with the Kantian doctrine seems to be that it maintains the infinite divisibility and infinite extent of time and space. The shade of Zeno with his hoary puzzles is invoked, and the awful consequences of infinite divisibility are vividly portrayed. To escape these consequences Professor Fullerton suggests that there are two kinds of space and time, real and perceptual.[1] Perceptual space, that given in intuition, is the actual space in which we move and with which we have our dealings. It is not infinitely divisible, but is composed of a finite number of *minima sensibilia.* Real space, on the other hand, is the space of mathematics, the space that is infinitely divisible, the space in which all Zeno's paradoxes hold without let or hindrance. This, however, need give us no concern; for it is not the space in which we have to move about. If it were there would be only one thing to say, we simply should not move for we should never get started. It is perfectly safe, however, to postulate this real space, since it, with all its difficulties, can be so conveniently disposed of in the sphere of mathematics, thus relieving ourselves of the embarrassment of trying to live in it.

Our examination of Professor Fullerton's position will fall under two heads. In the first place, we shall ask, does the Berkeleian doctrine really offer a solution for the difficulties raised; and second, is there any serious fallacy after all in the Kantian doctrine taken with its teaching as to infinite divisibility and infinite extension? We shall confine ourselves to the question of space since the solution of the time problem is necessarily very similar.

We maintain, in the first place, that the Berkeleian doctrine does not recognize or admit a real space such as Professor Fullerton sets up over against perceptual space, and that this division is made simply to escape absurdities in the Berkeleian doctrine that are even greater than the supposed difficulties in that of Kant. In the second place, not only does the Berkeleian theory not admit of these two varieties of space, but they are not even necessary or permissible in any consistent theory.

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[1] PHILOS. REV., Vol. X, pp. 595 ff, et al.

As is well known, Berkeley held that extension is composed of a fixed number of *minima visibilia* and *tangibilia* which are of unvarying dimensions.[1] Space as given in perception is a construct from the materials contributed by the several senses.[2] Visual space is nothing more than a definite number of *puncta visibilia* which, through shading and color, are symbolic of tactual space. Theoretically there is an absolutely separate space for each of the senses, but they are all perceptual spaces with exactly the same qualities of finite extension and finite divisibility. As sensible qualities they are all alike real. The *minima perceptibilia* are indivisible because it is impossible to assert reality of what does not enter into perception. "For, whatever may be said of extension in the abstract, it is certain sensible extension is not infinitely divisible. There is a *minimum tangible* and a *minimum visible* beyond which sense cannot perceive. This everyone's experience will inform him."[2] But abstract space, Professor Fullerton's 'real space,' he affirms over and over again is a mere invention. It is absurd to maintain an abstract extension that is infinitely divisible; what is too small to be discerned does not exist. Abstract extension is nothing.[3] Pure space is a negation.[4] Abstract space is a phantom of geometrical philosophers; it is not perceived by sense nor proved by reason.[5]

All space then, according to Berkeley, is perceptual, and there is nothing left for mathematics to do but to make the best of the conditions afforded by such an extension with its *minima puncta sensibilia*. Berkeley does not hesitate to point out some of the applications of his theory to mathematics. "Particular circles may be squared, for the circumference being given, a diameter may be found betwixt which and the true there is no difference . . . extension being a perception, and a perception not perceived is a contradiction. In vain to allege the difference may be seen by magnifying glasses, for in that case there is ('tis true) a difference perceived but not between the same ideas, but others much greater, entirely different therefrom."[6] Again, "The diagonal of a square is commensurable with its side, they both containing a certain number of *minima visibilia*."[1] The only proof for equal triangles is that they contain equal numbers of these visible points. Mathematical propositions must stand or fall on their own evidence, which is nothing more nor less than the evidence of perception.[2]

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[1] *Commonplace Book*, p. 471, Fraser ed. *Works*.

[2] *Works*, Vol. I, p. 94; *Commonplace Book*, p. 472.

[2] *Works*, Vol. I, p. 59.

[3] *Ibid.*, p. 221.

[4] *Ibid.*, Vol. III, p. 94.

[5] *Ibid.*, Vol. II, p. 468.

[6] *Commonplace Book*, p. 486.

We can readily see from the quotations given above that Berkeley would have been far from admitting the existence of Professor Fullerton's 'real space' made up of spatial elements too small to be perceived. It is, perhaps, unnecessary to quarrel about names, but it is surely misleading to call the doctrine that Professor Fullerton expounds the Berkeleian. The disparity is not so easily disposed of as he seems to think, by saying that the Berkeleian has only to admit that real space may be infinitely divided, and the Kantian, that real space is not given in intuition.[3] Unfortunately it is absolutely impossible for the Berkeleian to make this admission. His doctrine is perfectly consistent if we grant his premises. Reject the premises and try to set up a 'real space' by the side of the perceptual one and we are at once involved in all sorts of difficulties. The Berkeleian doctrine is of course unpalatable to any one with mathematical sensibilities, but the difficulties are not avoided by the setting up of a mathematical space for their accommodation. Berkeley realized this and so kept consistently to his perceptual space, bravely swallowing the mathematical pill, however bitter it might be.

The other suggestion, namely, that the Kantian admit that real space is not given in intuition, is as much out of the question as the first proposition. Kant, as well as Berkeley, stands for only a single space, whatever difficulties it may seem to involve. It remains for us to see whether this one space can really be described in a manner consistent with every-day experience, and as well with the necessities of mathematical science.

We may grant a mathematical space as merely an abstractly described perceptual space, but not one so materially different in quality as to permit within itself what is impossible in the latter. If motion along an infinitely divided line is really impossible, perceptual space is not freed from the dilemma by our maintaining that the infinitely little bits of line, perfectly admissible in mathematical space, are here unperceived. Such an argument is only a case of the ostrich annihilating its enemy by hiding its head in the sand, or of the pious Brahmin, who eased his soul by destroying the microscope that showed him that with every draught of water he destroyed countless numbers of living beings. It is not a solution of the difficulty to say that the moving body does not pass over an infinite number of positions, simply because these positions are not separately perceivable. If motion in mathematical space is impossible, as it surely is on Professor Fullerton's hypothesis, it must be impossible in perceptual space, whether we are able or not to count the infinite number of the stadia that lie in its pathway. We cannot say that since the elements of a series are too minute for the sense organs to distinguish them, they therefore do not exist, or may be disregarded in the actual motions of the world of intuition. The intuition may be too gross to admit of accurate description in mathematical terms, but, unless we accept the subjective idealism of Berkeley, we cannot maintain that the description in terms of immediate intuition is an adequate one.

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[1] PHILOS. REV., Vol. X, p. 487.

[2] *Ibid.*, p. 433.

[3] *Ibid.*, p. 385.

To every one but a Berkeleian, motion has an element of otherness about it, that forbids his regarding its description in perceptual terms as exhaustive. Perceptual motion is perhaps a passage from one *minimum sensibile* to another; but, from the mere otherness of motion, we are sure that it must have traversed extensions too minute to be perceived. In other words, the motion of perception is the motion of mathematical space described more grossly; and, in order that perceptual motion may exist for us at all, it must first have traversed the infinitely little bits of extension which Professor Fullerton admits constitute mathematical space.

To justify his conception of a line as made up of a finite number of *minima sensibilia*, Professor Fullerton maintains that in any experience the intuition of an object is built up out of many different elements contributed by the different senses, and that it is just so with the intuition of a line.[1] Now while the cases are not at all analogous, we may admit the truth of the conclusion in so far as it means that the intuition of a line is a construct, for this is nothing other than a point of Kant's with which Professor Fullerton in another place takes issue, namely, that to perceive a line we must draw it in imagination part by part. It is totally irrelevant to the discussion to insist that Kant meant the successive addition in infinite number of infinitely little bits of line, or mathematical points. He meant to say in absolutely generalized form that the process of perception is an active one, and in taking this position he offsets the apparent inadequacy of the standpoint he had set forth in the Æsthetic.

The *minima sensibilia* of the Berkeleian theory are of course unextended to perception. How then can they combine to produce extension? Professor Fullerton reasons from analogy that it is just as one line, though it does not make an angle, may yet do so if it is taken with another line, or that although one tree does not make a forest, many may do so. These analogies are utterly fallacious. They remind us of Hume's position that the idea of extension arises from colored and tangible objects having their parts disposed in a certain fashion. Hume completely begs the question by assuming a space in which to arrange his colored and tangible *minima.*[1] In just the same fashion two lines make an angle by virtue of relative position, and many trees make a forest, and many colored surfaces a variegated one, if we grant a space in which these trees and colored surfaces and lines may be disposed. But of the *minima sensibilia,* their only attribute is non-extension, and no adding of them together will produce extension. The fact is there is no such thing as a *minimum sensibile* that is unextended. Theoretically it may exist, perhaps, but here we are discussing the actual elements of perception and not those of some conceptual situation.

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[1] PHILOS. REV., Vol. X, p. 385.

If perceptual space is made up of *minima sensibilia,* he is in duty bound to prove why we see a line as a continuum instead of as an aggregate of these *minima,* or, as he himself says, why we are not clearly conscious of them as definite elements.[2] To admit that they are not as such present in consciousness, is to lay himself open to the error he attributes to Kant, of maintaining, as he assumes, that we have an intuition of infinitely divisible space.[3] Professor Fullerton shows that Kant had no right to assume the existence of an infinite number of parts outside of actual presentation. We waive the question as to whether Kant is fairly interpreted. The point is that Professor Fullerton does not refrain from a similar assumption in his argument for the existence of the *minima.* If his argument is correct he is surely far from any true Berkeleianism. The proof of the Berkeleian doctrine, then, is not *the direct reference to perception* that Berkeley himself maintained. Consciousness merely "seems to testify that any finite line is composed of simple parts."[4] In a finite line, so conceived, there is then only a finite number of possible positions. The only objection to assuming them infinite, is that such an assumption would, in Professor Fullerton's opinion, flatly contradict the well-attested fact of the motion of bodies through space. We shall see presently whether this is really the case.

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[1] See Hume's *Treatise* (ed. Green and Grosse), Vol. I, p. 194.

[2] PHILOS. REV., Vol. X, p. 385.

[3] *Ibid.*, p. 121.

[4] *Ibid.*, p. 385.

In concluding this part of the discussion, we may say that what we are actually dealing with is a continuum. It is not made up of parts at all, but in it there is the possibility of an infinite number of positions. The Berkeleian assumption is that space is not continuous, but is composed of a finite number of parts. The Kantian does not hold that extension, infinitely divisible, "consists in infinitely many parts." "All parts are contained in the intuition of the whole, yet the *whole division* is not contained in it," because it is the continuous decomposition that makes the series real.[1] This is equivalent to affirming that space, as actually presented, is continuous, and that infinite divisibility, instead of meaning an infinitely discrete space, means that there is no limit to the possible positions in space, if we wish to look for them.

Before inquiring into the real meaning of space, infinitely extended and infinitely divisible, it will be wise to clear up our several uses of the term infinite. It may mean unending extension in all directions, or unending extension in only one direction from a given point, or, lastly, it may be used to describe the number of positions in a given finite distance. As there may be any number of finite lines this would seem to indicate that there may be any number of differently sized infinities, a manifest contradiction in terms. These uses of the term make it evident that it is purely a limiting notion, and that it does not admit of translation into perceptual language.

Infinite in the first sense, to the ordinary man, is a descriptive term referring to the magnitude of the external world of time and space. To one who seeks to define the idea more accurately, it is simply the expression of the fact that *all our knowledge of things falls ultimately into spatial and temporal terms.* It matters little whether we say that space and time are necessary forms of intuition; it is at least certain that they *are* the forms of our present perceptions. It is perfectly legitimate for Kant to say that space cannot be represented as nonexistent, although the objects in space can be so disposed of. Kant raises the question simply to show that it is a contradiction in terms. Professor Fullerton rightly ridicules Hamilton and others for supposing they have proved the infinity of space by saying that we cannot think of space as limited by boundaries beyond which there is no space. The fallacy in such reasoning is, of course, the assumption of space as existing independently of a perceptive consciousness. If space is thus a something apart, it is absurd to conclude that it is infinite because we cannot think of it as bounded. The realist apparently conceives his infinite space as built up by the successive adding, without end, of part after part to his present patch of intuited space. It is manifestly impossible for him, therefore, to make dogmatic assertions about what may or may not be true after indefinite additions have been made to this realistic space. His present inability to think an end to the process is absolutely no proof that there is none. His space is an independent quasi-entity, and as such he has no right to make sweeping statements about it.

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[1] *Critique of Pure Reason*, Müller's Trans., Vol. II, p. 453.

With Kant, however, the case is different, for he stands on different ground. When he speaks of infinite space he does not mean what the realist does by the same expression, although Professor Fullerton apparently interprets him so. Kant does not foolishly announce that we are unable to think of a bounded space in the midst of an unextended. That is a platitude. Instead, he shows us that all our world of phenomena is spatial and temporal. Everything that is to be experienced by us as an object must be spatially determined; hence, from Kant's point of view, to *represent* space as non-existent is a contradiction in terms. If space is a form of intuition, Kant can say with absolute certainty that it is infinite, but not, of course, in the crude sense of the realist. Professor Fullerton maintains that the intuition of space is limited to the intuition of the object; that it is impossible to extract an intuition of infinite space from the patch of sensation with which we start. (Note that Kant does not use intuition, *Anschauung*, in this connection but *Vorstellung*, or representation.) By infinite space Professor Fullerton evidently has in mind a very big something that, if it can be said to exist at all, is *so* big that it cannot be grasped in a single intuition. Kant, however, may mean by infinite, something very different from this, and hence may well be untouched by his ridicule.[1] What Kant actually says is: *Der Raum wird als eine unendliche gegebene Grösse vorgestellt.* Professor Fullerton takes him to mean that "we are intuitively conscious of every part of space as we are conscious of the bit of space within the limits of this patch of sensation."[2] To urge this objection against Kant's *Vorstellung* of the infinite, is not merely to quibble on *Vorstellung*, but, worse than that, it shows a total misapprehension of what that philosopher meant by infinite. If he had meant it in the old realistic sense, we should not hesitate long over the question of rejecting his doctrine. We have a

[1] PHILOS. REV., p. 118.

[2] *Ibid.*, p. 117.

representation of only so much of this realistic space as is given in our present tactual and visual fields. Now, when the merest patch of space around the corner is evidently not in our immediate experience, it is hardly conceivable that Kant should have made the glaring blunder of maintaining that infinite space, in this sense, is a *Vorstellung*.

After interpreting Kant as though he were a realist, Professor Fullerton tries to mitigate the absurdities in which he imagines the former is involved by the suggestion that Kant's intuition (really *Vorstellung*), of infinite space is not intuition in the sense of perception, but in the other English sense, that is, a sort of feeling for a thing that goes without definite proof. Here there is surely a confusion. First, intuition (*Anschauung*) is substituted for representation (*Vorstellung*), and then the *dénouement* is accomplished by a sort of *deus ex machina* of linguistic subtlety, whereby an English idiomatic meaning is read back into the German *Anschauung*.

It may be noted also, in passing, that Kant maintains that space is a necessary form of our *perception* of the world of objects, not a necessary form of *thought*, as Professor Fullerton over and over again represents. Thus he gives us the ingenious quibble on page 118 as a point in his argument against Kant. He says, in substance, that Kant regards space as a necessary form of thought; but space as a mere form of thought, as well as its content, can be thought away. Hence space is not a necessary form of thought at all. But space, with Kant, is a form of the intuition, or perception, and, as such, he tells us we can never represent to ourselves its non-existence. It is manifestly impossible to intuit the end of intuition, or to perceive the world in the midst of the imperceptible. To be able to do so would be to make the imperceptible in a sense perceptible, that is, spatial. We cannot destroy the conditions of sensibility without destroying sensibility itself.

This brings us to the question as to what Kant really meant by his statement that space is represented as an infinite given quantity. Evidently he must be interpreted in terms of his general point of view. We have anticipated in the preceding paragraph the beginning of the answer. The representation of infinite space is not the representation of an infinite series of finite spaces, as Kant might have meant if he had been a realist. It is a presupposition of the present experience and is involved in it. If space is a necessary form of sensibility, the presentation of an object contains within itself a significance beyond the mere intuition. An intuition is an act in an entirely closed system, and its present meaning is not felt to be adequately expressed unless this fact is recognized. When the common-sense man claims that he can imagine endless extension, because he can image this present object and then another and another, does he really mean anything other than what we have stated above? He feels his present intuition in a certain relation or setting. This relation is a part of the given intuition; each succeeding intuition shares in the same relation. The representation (*Vorstellung*) of the absolute and all-inclusive significance of the space world is thus present implicitly in every perception. Every recognition of this or that object in space is a recognition of the system of relations in which it is set. The representation of infinite space is thus a part of every analyzed spatial presentation. To understand the full significance of this, we must inquire more carefully into what Kant meant by infinite. To apply this term to our space world is only another way of asserting that it is our only world. It is just because of its absolute uniqueness that it can be described in no other way than as infinite. We cannot represent to ourselves an end to the possibility of positions within it. Infinity is then simply the quantitative way of saying that our world is a continuous one, a closed system. When Kant says space is infinite, he really says nothing about an endless series of extensions, quasi-independent of consciousness. He simply states the significance of the elements of the series in order for them to be elements at all; just as when we say two parallel lines meet at infinity, we think of the term as significant of the relation of finite elements of the lines and not of their length. This is not, of course, Professor Fullerton's use of infinite, but it is evidently what Kant meant when he used the term.

Summarized briefly, the above discussion comes to this: Kant meant by space the *form*, or *law* of intuiting as well as the *product*, or *intuited* space.[1] It is the presence of the former element in every space perception which Kant has in mind when he says that space is represented as an infinite given quantity. We grant the right to question whether he is justified in his use of the term representation in these cases, but we demand that what he does say be interpreted in the light of his general standpoint, and not as though they were the expressions of a realist.

That infinite is thus really a limiting notion, a term expressing our relation to the world of phenomena, and hence a term that cannot be used in the old realistic sense as characteristic of the world as an existence apart from consciousness, Kant shows conclusively in the first and second Antinomies. Infinity is not a term that can be applied to space as a thing in itself; of such a space nothing can be said. If we try to characterize it, we find ourselves in the dilemma of having two mutually contradictory propositions on our hands, each apparently equally capable of proof. It is because Professor Fullerton unconsciously assumes a realistic space, that he sees in the Antinomies only hopeless confusion for all advocates of the infinity and infinite divisibility of extension.

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[1] *Critique of Pure Reason*, 2d ed., p. 160.

So much for Kant's conception of infinite as applied to the extent of space. But what of infinite divisibility? This brings us to the other meaning of the term that we set out to examine.

Does motion from one point to another mean that an infinite series of positions must be traversed, however small the separating finite distance? Exactly so, for as stated above, an infinite series of positions between two points is exactly equivalent to a continuum between the points. It is a contradiction in terms to deny the possibility of motion in a continuum, on the ground that the series of positions to be passed over is infinite. Such a statement assumes that what is without limit must be brought under the category of limits, if motion is to be possible, and, since the first part of the hypothesis is impossible, motion is declared to be out of the question. Upon the limiting category of infinite is imposed the perceptual category of number; and because they are found to be incompatible we deny that the space of perception is capable of infinite division. The conception of infinite that we seem to have in mind is again that of a very big series of numbers, whereas in truth it is merely a term by which one continuum can, without reference to any other, larger or smaller, be placed in the category of quantity; that is, a continuum can thus, in and of itself, be described as an aggregate.

To assert, however, that when a continuum is traversed the elements of an infinite series must be *numbered*, is nothing else than applying finite concepts to what is admitted to be beyond the pale of the finite. Suppose the unending process of enumeration does take place; if it does, it will by hypothesis require an endless time. But, in assuming such a process, we must remember that we are no longer dealing with a space at all but are trying to combine in an ideal perceptual experience two mutually irreconcilable categories, namely, that of numeration, and hence of limitation, with that of infinity or the unlimited.

Motion for any assignable distance passes over an infinite number of intermediate positions, but it does it in a finite time. It is perfectly legitimate, as against Professor Fullerton, to say that the moving body has an infinite number of infinitely little bits of time with which to make its paradoxical journey. But this means nothing more than that there are given coincidentally three continua, the finite line, a finite time, and a finite motion as the content of the time. These three continua, expressed quantitatively, may be said to contain an infinite number of infinitely small parts, but, as we have said repeatedly, this means nothing other than that these so-called aggregates are continua. It is absurd, then, and just here is the solution of Zeno's puzzle, to insist that an infinite number of infinitely little bits of line must be traversed in an infinite number of finite bits of time. This clearly would be an endless process. If Clifford and Professor Fullerton had used the same intellectual shorthand for both the time and space series, the paradox of the moving point would never have troubled them.

If we conceive a continuum as an aggregate of infinitely minute elements, it is manifestly absurd to inquire what lies between these elements; and yet this is what the theorists of Clifford's type tacitly assume must be done. By hypothesis the term infinite is a limiting one, the limit toward which an aggregate may proceed; but when it has arrived at the limit, it ceases to be an aggregate and becomes a continuum, hence the idea of a 'between' is excluded. When we speak of these infinitesimal elements we must constantly bear in mind that they do not permit of translation into perceptual terms. They are symbolic of the limits to which finite experience can be carried back. Any finite distance, though indefinitely small, may be ideally perceived, but when the limit, infinity, is reached we have passed beyond the possibility of imagery. Hence the question as to how an infinite number of these infinitesimals can make a finite line is totally irrelevant. To ask the question means that we are trying to image the limit, or, in other words, the continuum as discrete. Those who assert that motion over an infinitely divided line is impossible in a finite time seem to think of the line as composed of mathematical points separated by finite distances, thus assuming a space back of, and as a setting for, the space is question. The paradox of Zeno, then, arises out of the assumption that an infinitely divided line is discrete. The arguments of Zeno, in fact, if correctly interpreted actually reënforce our theory. Tannery[1] shows almost conclusively that Zeno's criticisms were directed against the small discrete elements of the Pythagoreans and not against the popular ideas of motion. His arguments showed the absurdity of the Pythagorean hypotheses, which, if carried out consistently, would result in the denial of the possibility of motion.

[1] *Pour l'histoire de la science hellene,* Chap. X, pp. 248 ff.

Instead, then, of Zeno forcing us to such a doctrine as Berkeley's, he offers us the best of reasons for avoiding it.

We may in conclusion devote a few words to Professor Fullerton's revolving disk.[1] He argues that if a point can traverse an infinite series of points in one second, then also the speed of a revolving disk may be increased by geometrical progression until at the end of one second its rate of rotation will be infinitely great. In brief, we may say that the moving point and the revolving disk involve two totally different problems. In the first case, we have a diminishing series of spaces, in the latter, an increasing series of velocities. In the second case, also, it is assumed that an infinite number of finite lengths, viz., the circumference of the disk multiplied by infinity, is traversed in one second. We agree with Professor Fullerton that it is absurd to suppose that an infinite series of finite distances can be covered in a finite time; but this is not the problem presented by the point moving along the finite line. Here we have a series of infinitely little spaces covered in a finite time, in one second by the example, and it is perfectly conceivable because they are infinitely little and hence no spaces at all. If Professor Fullerton had made the size of his disk diminish to a point in one second, at the same time that its velocity was increasing to infinity, the paradox would have vanished in more ways than one, and the two cases would have been parallel.

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[1] PHILOS. REV., Vol. X, p. 235.